RESEARCH DEPARTMENT

RECEIVER SIGNAL/NOISE RATIO IMPROVEMENT USING POST DETECTOR SELECTIVITY

Report No. G-057

(1955/17)

THE BRITISH BROADCASTING CORPORATION

ENGINEERING DIVISION

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SUMMARY

In the design of radio receivers for the measurement of certain types of amplitude-modulated transmission, a useful increase in the output signal/noise ratio can be achieved by restricting the bandwidth after detection. Even when the signal is well below noise level at the detector input, it may be considerably above noise level at the output of the receiver. The effective gain of the receiver will then, however, depend on the signal/noise ratio at the detector input. The change of gain resulting from the insertion of fixed attenuators before the detector will therefore vary with the output signal level and a new input/output calibration will be required for each value of attenuation.

The results of previous theoretical analyses for an idealised linear detector followed by a low-pass filter are given. The theory is extended to apply to the detection of square-wave modulated signals using a band-pass a.f. filter followed by a second detector. The results of tests on one particular receiver show reasonable agreement with this theory.

SYMBOLS USED IN TEXT

v_i	r.m.s. signal voltage at input to receiver
v_s	r.m.s. signal voltage at input to i.f. detector
v_n	r.m.s. noise voltage at input to i.f. detector
i_{dc}	total d.c. output of i.f. detector
i^o_{dc}	d.c. output of i.f. detector in absence of signal
i_s	increase in d.c. output of i.f. detector due to presence of signal
i_m	r.m.s. current output of i.f. detector at modulating frequency f'_o
i'_{dc}	total d.c. output of a.f. detector
f_o	intermediate frequency
Δf	noise bandwidth of i.f. amplifier, centred on f_o
f'_o	modulating frequency
$\Delta f'$	noise bandwidth of a.f. filter, centred on f'_o
s	r.m.s. signal/noise ratio at input to i.f. detector
$C(s)$	function relating i_{dc} to s for a linear detector
$D(s)$	function relating P_s to s for a particular receiver
$F(s)$	function relating i'_{dc} to s for a particular receiver
S_o	r.m.s. ratio of signal output from a.f. filter to noise output, the noise being measured in the absence of signal
S_s	r.m.s. ratio of signal output from a.f. filter to noise output, the noise being measured in the presence of signal
S_{mo}	S_o } for a signal square-wave modulated at f'_o and a band-pass a.f. filter of width $\Delta f' << f'_o$
S_{ms}	S_s }
p_o	power density of noise spectrum at input to i.f. detector
P_o	power density of noise spectrum at output near $f = 0$ in absence of signal
P_s	power density of noise spectrum at output near $f = 0$ in presence of signal
P_m	mean value of P_s for modulated signal
N	overall noise factor of receiver
N_o	overall noise factor of receiver with no i.f. attenuator
A	insertion loss of i.f. attenuator
R_n	reduction in r.m.s. noise voltage at i.f. detector on inserting attenuator

1. INTRODUCTION.

The accurate measurement of the amplitude of a small variable signal requires a receiver having sufficient gain, no internal sources of thermal noise and a bandwidth only just sufficient to pass the required signal information. In practice, the maximum useful gain is limited by the presence of thermal noise sources; these add fluctuations to the signal which reduce the accuracy of each observation, the accuracy depending on the r.m.s. signal/noise ratio at the output of the receiver. If this ratio is to be as large as possible, all those noise components having frequencies not occupied by the signal must be excluded from the detector. This can be done in some systems, such as communications links, by restricting the bandwidth of the circuits preceding the detector to that occupied by the signal information. But in a system designed for the measurement of relatively slow variations in the amplitude of a signal this may be impossible, due to the frequency drift of the transmitter relative to the receiver. The pass-band of the receiver must then be sufficiently wide to accommodate the maximum frequency drift and, for a given minimum signal input, the extra noise components included will affect the accuracy of measurement.

These unwanted noise components can be reduced by restricting the bandwidth of the circuits following the detector. The improvement with the conventional types of incoherent detector is less than would be achieved by a similar restriction of the i.f. bandwidth; this is due to intermodulation of the unwanted components in the detector. A practical detector of this type is difficult to represent theoretically but its main characteristics are thought to be represented by the so-called ideal linear detector. Bennett[1] has derived theoretical expressions for the behaviour of such a device, taking account only of first-order intermodulation products of signal and noise. He has shown that a closer approximation may lead to an increase of about 0·5 dB in the calculated noise output, giving closer agreement with experimental results. Using the first-order approximation, Smith[2] has derived expressions for the theoretical improvement in signal/noise ratio using a low-pass a.f. filter, and his results are given in Section 2. The theory is extended in Section 3 to apply to a particular system using a band-pass a.f. filter and a square-wave modulated signal.

An important feature of the linear detector is that, for a small input signal, the signal output is inversely proportional to the noise input; this effect is discussed in Section 3.4 with special reference to the use of attenuators in the i.f. amplifier of the receiver.

2. THEORETICAL RESULTS FOR AN IDEALISED SYSTEM.

2.1. Behaviour of the Detector.

Consider an idealised system, which might be approached in a practical receiver, consisting of an ideal linear detector preceded by a band-pass filter with a rectangular response of bandwidth Δf, centred on f_0. We shall initially consider this to be followed by a low-pass filter which removes input frequencies and their harmonics but passes all frequencies up to $2\Delta f$.

The detector may be represented by the expression

$$\left.\begin{aligned} i &= bv \quad v > 0 \\ &= 0 \quad v \leqslant 0 \end{aligned}\right\} \tag{1}$$

where i and v are the instantaneous values of the output current and input voltage respectively.

When a sinusoidal signal of r.m.s. voltage v_s is applied to such a device, the d.c. output is

$$i_{dc} = \frac{\sqrt{2}\, b}{\pi} v_s \tag{2}$$

For convenience the detector load is assumed to be unity; i_{dc}^2 therefore represents the total power output of the detector since all harmonic currents are returned by a low impedance path.

If, instead of a signal, a normally distributed noise voltage of r.m.s. value v_n is applied, the d.c. output will be

$$i_{dc} = \frac{b}{\sqrt{2\pi}} v_n \tag{3}$$

Here, the d.c. power output i_{dc}^2 represents only a part of the total power output, the remainder being delivered in a low-frequency noise spectrum generated by intermodulation of components of the input spectrum and extending from $f = 0$ to $f = \Delta f$, as shown in Fig. 1(a)(ii).

If a steady signal at f_0 is added to the noise input, the d.c. output may be expressed as the sum of two terms,

$$i_{dc} = i_{dc}^{o} + i_s \tag{4}$$

where i_{dc}^{o} is the output due to noise alone and i_s is the increase in d.c. output due to the presence of the signal. The signal output is now no longer a linear function of the signal input; the relationship is described in Section 2.2. The output noise spectrum contains additional components generated by intermodulation of signal and noise and extending from $f = 0$ to $f = \frac{1}{2}\Delta f$; the result is shown in Fig. 1(b)(ii). The output noise power density in the pass-band of the post-detector filter is given in Section 2.3.

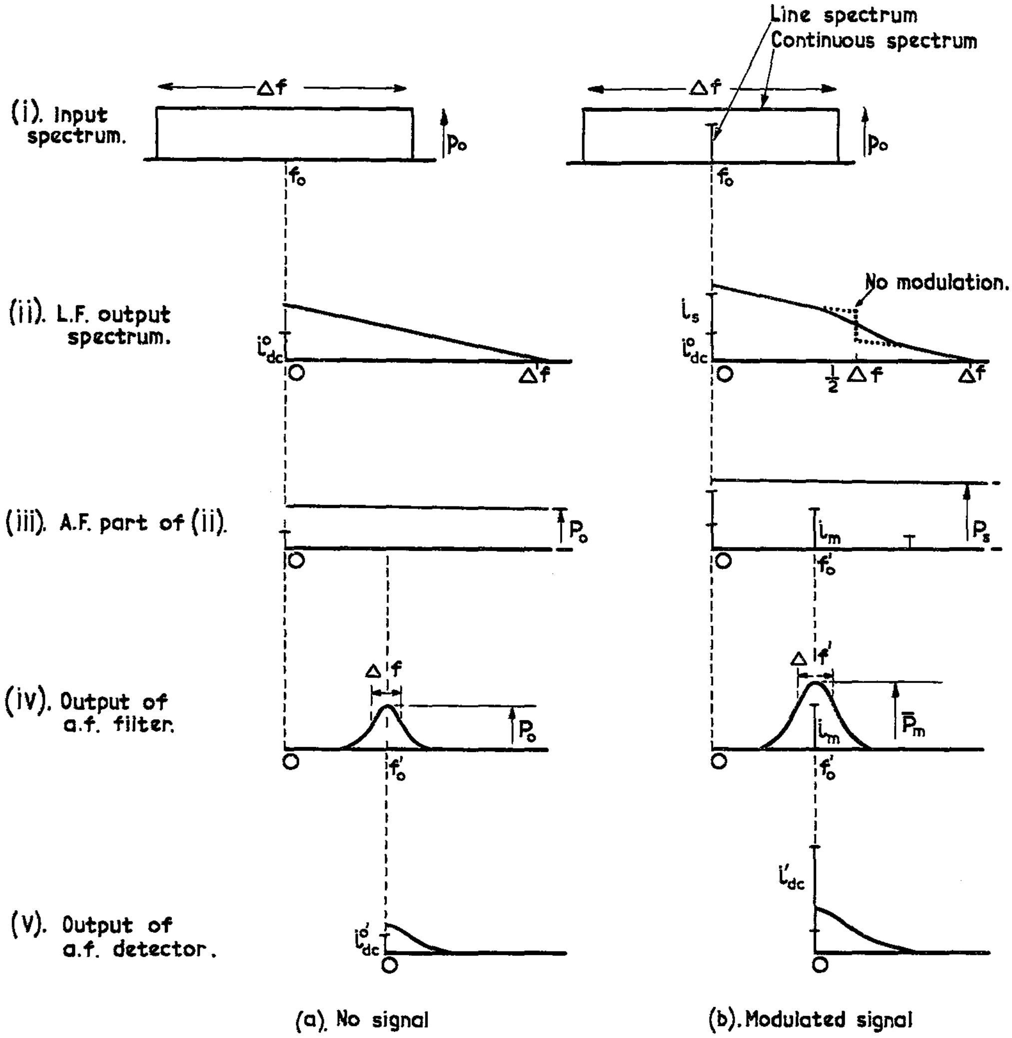

Fig. 1 - Spectra of noise and signal

2.2. D.C. Output.

The d.c. output of the linear detector, in terms of the r.m.s. noise input, v_n, and the input r.m.s. signal/noise ratio $s = v_s/v_n$, is

$$i_{dc} = \frac{b}{\sqrt{2\pi}} v_n \, C(s) \tag{5}$$

where $C(s)$ is a function of s which is plotted in Fig. 2.

$$C(s) \to 1 + \tfrac{1}{2} s^2 \quad \text{as} \quad s \to 0$$

$$\text{and } C(s) \to \frac{2s}{\sqrt{\pi}} \quad \text{as} \quad s \to \infty$$

Equation (5) reduces to equations (2) and (3) for large and small values of s, respectively. Substituting in equation (4) we have

$$i_s = \frac{b}{\sqrt{2\pi}} v_n \left[C(s) - 1\right] \tag{6}$$

$$\rightarrow \frac{b}{\sqrt{8\pi}} v_n s^2 \quad \text{as} \quad s \rightarrow 0$$

Thus, the linear detector behaves as a square-law detector for low input signal/noise ratios.

It is interesting to note, from Fig. 2, that for $s = 1$, $C(s) = 1{\cdot}44$, representing an increase in d.c. output of 3·2 dB due to the addition to the noise of a signal of the same r.m.s. value. The resemblance to r.m.s. addition is, however, fortuitous; the subsequent removal of the noise results in a reduction in the d.c. output of only 2·2 dB.

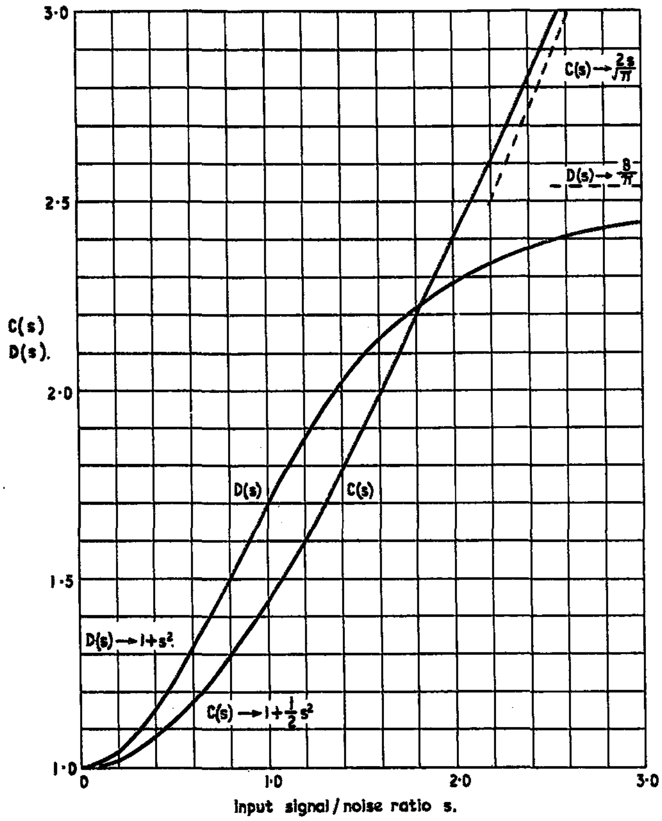

Fig. 2 - The functions C(s) and D(s)

2.3. Noise Output.

For an input filter with a rectangular pass-band of width Δf, the r.m.s. input noise voltage is

$$v_n = \sqrt{p_o \Delta f} \tag{7}$$

where p_o is the power density of the input noise spectrum. The power density of the output spectrum at frequencies near $f = 0$, where the post-detector a.f. filter is to be situated, is

$$P_s = \frac{b^2 p_o}{4\pi} D(s) \tag{8}$$

where $D(s)$ is a function of s which is plotted in Fig. 2. For small values of s, $D(s) \rightarrow 1$ and for large values, $D(s) \rightarrow 8/\pi$. The value of P_s in the absence of signal is then

$$P_o = \frac{b^2 p_o}{4\pi} \tag{9}$$

and the noise power transmitted by a low-pass filter of bandwidth $\Delta f' << \Delta f$ in the

presence or absence of signal is obtained by multiplying equations (8) and (9), respectively, by $\Delta f'$. The form of these output spectra can be seen from Fig. 1(iii) and (iv).

2.4. Output Signal/noise Ratio.

Expressions for the signal and noise output have been given in equations (6), (8), and (9). The noise output rises when a signal is applied; Smith[2] has therefore defined the ratio of signal output to the noise output in both the presence and absence of signal. These are, respectively,

$$S_s = \frac{i_s}{\sqrt{P_s \Delta f'}} \tag{10}$$

$$\text{and } S_o = \frac{i_s}{\sqrt{P_o \Delta f'}} \tag{11}$$

Each of these quantities forms a useful basis for comparison of various types of detector but neither represents the a.f. signal/noise ratio required here, which will depend on the modulating function superimposed on s and its effect on the output noise power. We shall, in the next section, obtain an approximate value for the output signal/noise ratio in a particular receiver when the input signal is modulated by a square wave.

3. APPLICATION TO A PARTICULAR RECEIVER.

3.1. Experimental Details.

A high-stability field-strength recording receiver[3], designed in Research Department, has a signal-frequency response within ± 0·2 dB over a bandwidth of 400 kc/s, and an energy bandwidth of about 500 kc/s. The transmitted signal used with this receiver is 100% modulated with a 1000 c/s square wave. The 10·7 Mc/s i.f. amplifier is followed by a diode detector, the output of which is passed through a simple filter, resonant at 1000 c/s and having an energy bandwidth of about 30 c/s; it is then amplified and rectified in an a.f. bridge detector with negative current feedback. The d.c. component of the output of this second detector is recorded on a paper chart, and a calibration refers the recorded current to the signal input.

By means of automatic gain control a recording range of about 60 dB is achieved, that is, from 40 dB above noise level to 20 dB below noise. To extend this range to higher input levels, resistive attenuators may be inserted before the i.f. amplifier; but, as will be shown from the foregoing theory, a new calibration is required for each attenuator.

3.2. Measured Signal/noise Ratio Improvement.

A test was made on this receiver to compare the signal input required for an a.f. signal/noise ratio of unity with that required for the same ratio in the i.f. amplifier. The measurement was made using a calibrated piston attenuator and a thermocouple; the improvement ratio was found to be −18·5 dB.

3.3. Theoretical Signal/noise Ratio Improvement.

If the signal input is modulated, the output spectrum will be modified as indicated by the solid line in Fig. 1(b)(ii). Considering only first-order intermodulation products, a low-frequency modulation will only disturb the output spectrum in the vicinity of $f = \frac{1}{2}\Delta f$, but if the modulation frequencies extend up to $\frac{1}{2}\Delta f$ the disturbance of the spectrum will extend down to the region near $f = 0$. The spectrum of the square-wave modulation used in this receiver varies as the inverse square of the frequency and the selected fundamental frequency f_0' is very small compared with Δf. The shape of the output spectrum may therefore be treated as if the signal power were all concentrated at f_0; thus we need only consider the behaviour of the detector in the presence and absence of a steady signal of amplitude equal to that of the peak input.

If the r.m.s. value of the modulated input signal/noise ratio is s, it must be alternating during the modulation cycle between zero and $\sqrt{2}s$, and the peak value of the change in d.c. output due to the signal will be, from equations (6) and (7)

$$i_s = b\left[\frac{p_0 \Delta f}{2\pi}\right]^{\frac{1}{2}} [C(\sqrt{2}s) - 1] \qquad (12)$$

The fundamental component of the modulation at f_0' will have a peak-to-peak amplitude of $4/\pi$ times greater than this, and its r.m.s. value will be

$$i_m = \frac{\sqrt{2}}{\pi} i_s \qquad (13)$$

The output noise spectrum will be modulated by the signal but, since the a.f. filter has a bandwidth $\Delta f' \ll f_0'$, the noise power transmitted by it will not vary during the modulation cycle; from equation (9), the mean value is

$$P_m \Delta f' = b^2 \frac{p_0 \Delta f'}{4\pi} \left[\frac{1 + D(\sqrt{2}s)}{2}\right] \qquad (14)$$

in the presence of the modulated signal.

Hence, the effective r.m.s. output signal/noise ratios corresponding to those of Smith in equations (10) and (11) are

$$S_{ms} = \frac{2\sqrt{2}}{\pi}\left(\frac{\Delta f}{\Delta f'}\right)^{\frac{1}{2}} \frac{C(\sqrt{2}s) - 1}{[1 + D(\sqrt{2}s)]^{\frac{1}{2}}} \qquad (15)$$

$$S_{mo} = \frac{2}{\pi}\left[\frac{\Delta f}{\Delta f'}\right]^{\frac{1}{2}} [C(\sqrt{2}s) - 1] \qquad (16)$$

Equation (15) represents the ratio of signal output to that noise output occurring in the presence of signal, and equation (16) gives the ratio of signal to that noise output occurring in the absence of signal; these are shown in Fig. 3. The experimental result obtained in Section 3.2 was for the latter quantity, the total power output being observed with and without the signal.

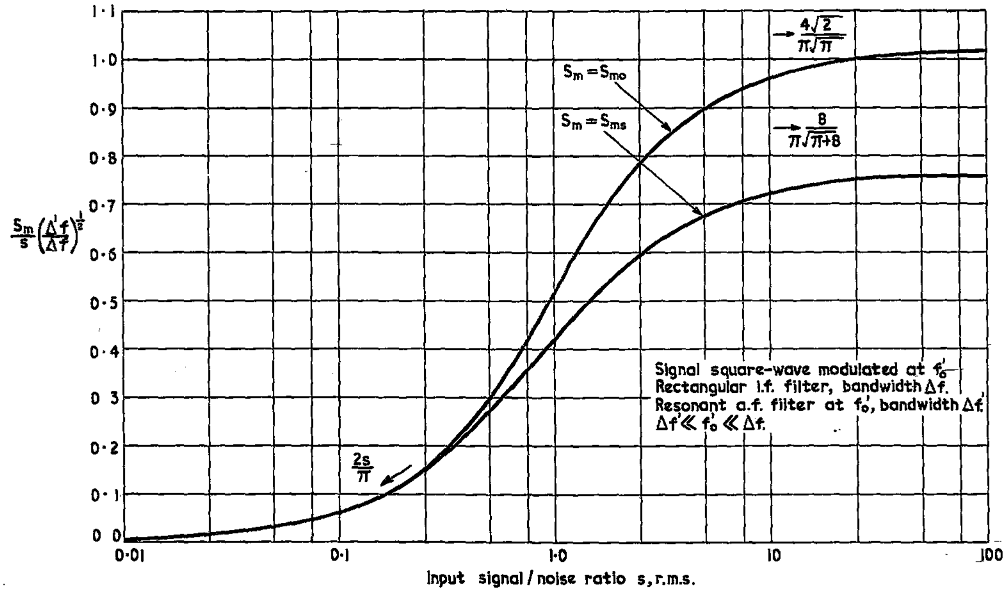

Fig. 3 - Output signal/noise ratios S_{mo}, S_{ms}

To check the experimental value we may use the expressions for small values of s and write

$$\left.\begin{aligned} C(\sqrt{2}s) - 1 &= s^2 \\ D(\sqrt{2}s) &= 1 \end{aligned}\right\} \qquad (17)$$

giving, for small values of s,

$$S_{mo} = S_{ms} = \frac{2}{\pi}\left\{\frac{\Delta f}{\Delta f'}\right\}^{\frac{1}{2}} s^2 \qquad (18)$$

Inserting the values of $\Delta f = 5.10^5$ c/s, $\Delta f' = 30$ c/s, and putting $S_{mo} = 1$, we obtain the input signal/noise ratio $s = 0{\cdot}11$, or $-19{\cdot}2$ dB; this is in fairly close agreement with the observed improvement ratio $-18{\cdot}5$ dB.

3.4. Distortion of Calibration by I.F. Attenuators.

A peculiarity of the linear detector, when working at a low signal/noise ratio, may be illustrated by referring to equation (6),

$$i_s \rightarrow \frac{b}{\sqrt{8\pi}} v_n s^2 \quad \text{as} \quad s \rightarrow 0 \qquad (6)$$

This may be written

$$i_s \propto \frac{v_s^2}{v_n} \quad \text{for} \quad v_s \ll v_n$$

showing that, for small signals, the signal output is inversely proportional to the noise voltage applied to the detector.

If, for example, a 10 dB attenuator is inserted before the detector, both v_s and v_n are reduced by the same amount and the fall in signal output is also 10 dB. But to raise the output by 10 dB requires an increase in v_s of only 5 dB. Thus, the signal input to the receiver must be changed by 5 dB to compensate for the insertion of a 10 dB attenuator.

This effect is, of course, present without post-detector filtering but its importance here lies in the fact that the signal is below noise level at the detector input over a large part of the working range.

In the receiver described in Section 3.1, an approximately logarithmic input/output law is achieved by using a.g.c., so that the gain of the i.f. amplifier varies as a function of the recorded output. To find the change in this law due to the addition of an attenuator, without defining the a.g.c. characteristic, we must find the change in the signal/noise ratio in the i.f. amplifier required to maintain a constant output from the a.f. detector at each point of the calibration.

Using the same functions as for the i.f. detector in equation (5), the d.c. output from the a.f. detector is, from equations (14) and (15),

$$i'_{dc} = b' \left[\frac{P_m \Delta f'}{2\pi} \right]^{\frac{1}{2}} C(S_{ms}) \tag{19}$$

which may be expressed in terms of the i.f. signal/noise ratio by

$$i'_{dc} = K \, v_n \, F(s) \tag{20}$$

where

$$K = \frac{bb'}{\sqrt{8\pi}} \left[\frac{\Delta f'}{\Delta f} \right]^{\frac{1}{2}}$$

a constant,

$$v_n = (\phi_o \Delta f)^{\frac{1}{2}}$$

the r.m.s. noise input to the i.f. detector

$$\text{and } F(s) = C(S_{ms}) \left[\frac{1 + D(\sqrt{2}s)}{2} \right]^{\frac{1}{2}}$$

The function $F(s)$ has been calculated for the receiver under discussion, and is plotted in Fig. 4.

If an attenuator is inserted at some point in the receiver, the r.m.s. noise input to the detector will be reduced by a factor

$$R_n = \frac{v_{no}}{v_n} \tag{21}$$

where the suffix "o" refers to the absence of the attenuator. For any given output

current i'_{dc} it follows from equations (20) and (21) that

$$F(s) = R_n \, F(s_o) \qquad (22)$$

using this result and the plotted function $F(s)$ shown in Fig. 4, it is possible to derive the change in input signal/noise ratio required to maintain a given output current when the attenuator is inserted.

Taking a practical example, let us consider the effect of inserting a 20 dB attenuator between the i.f. amplifier and the signal-frequency unit. The attenuator, as well as reducing the input signal and noise by a factor A, its insertion loss, also increases the noise factor of the receiver, N, due to the presence of noise sources in the i.f. amplifier.

The noise factor of the receiver is

$$N = N_1 + A \frac{N_2 - 1}{G_1} \qquad (23)$$

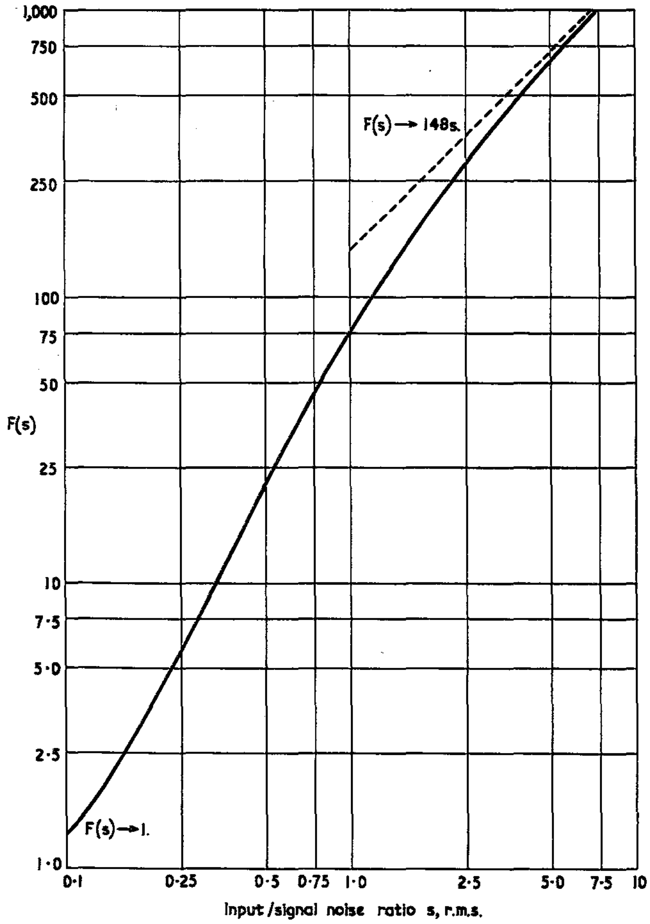

Fig. 4 - The function F(s) for a particular receiver

where N_1 = Noise factor of signal-frequency unit

G_1 = Power gain of signal-frequency unit

A = Insertion loss of attenuator

N_2 = Noise factor of i.f. amplifier

With the receiver under consideration the values were:

N_1 = 5·0 (7 dB); N_2 = 2·2 (3·5 dB); G_1 = 32 (15 dB)

The reduction in the noise factor is therefore given by

$$R_n = A \frac{N_o}{N} \qquad (24)$$

and the following table gives the values for each value of A.

Attenuation	Noise factor, N	Increase in N	I.F. noise reduction factor R_n	
dB	dB	dB	dB	r.m.s.
0	7·0	0·0	0·0	1·0
20	9·3	2·3	17·7	7·7

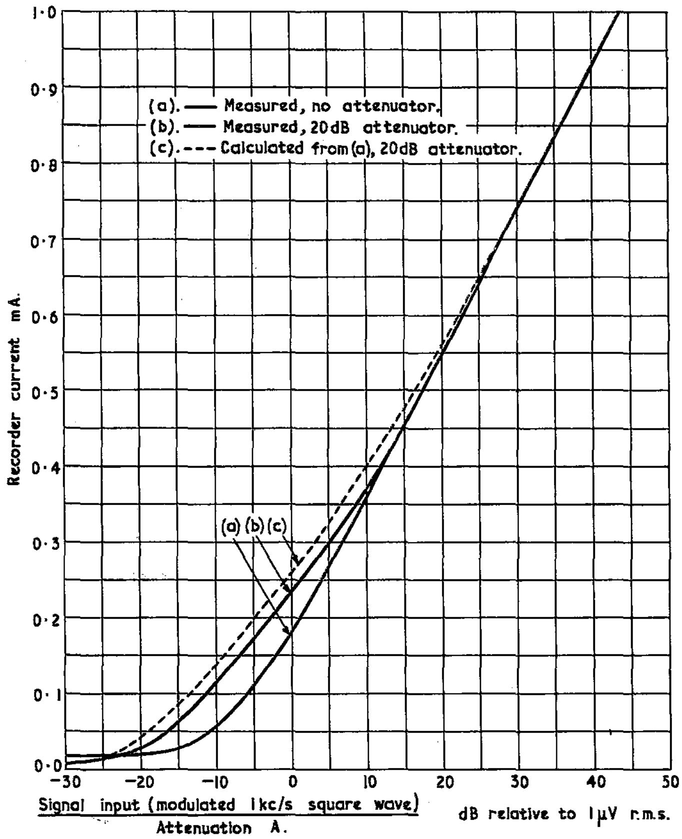

Fig. 5 - Calibration curve for a particular receiver, showing distortion by attenuator

Typical calibration curves for this receiver are shown in Fig. 5; these show the recorder output current i'_{dc} as a function of the signal-frequency r.m.s. input voltage v_i. For convenience this input voltage is shown divided by the loss of the attenuator, so that the abscissa is proportional to the detector signal input, v_s, for a constant value of output current. The curves (a) and (b) were found experimentally using 0 dB and 20 dB attenuators, showing a change of up to 10 dB in the calibration over a third of the recording range.

Curve 5(a) was used to find the input signal/noise ratio, using the expression for the r.m.s. noise voltage referred to the input of the receiver,

$$s_o = v_i (4 k T \Delta f R N_o)^{-\frac{1}{2}} \qquad (25)$$

where k = Boltzmann's constant

T = Temperature °K

Δf = Energy bandwidth

R = Source resistance presented to receiver

N_o = Noise factor of receiver with no attenuator

Knowing s_o and R_n, the value of s was found, using equation (22) and Fig. 4; the change of signal level for constant output was obtained from

$$\frac{v_s}{v_{so}} = \frac{1}{R_n} \frac{s}{s_o} \qquad (26)$$

and a curve plotted for the 20 dB attenuator, Fig. 5(c). The experimental result for a 20 dB attenuator, curve (b), shows qualitative agreement with the theory, though there is a discrepancy of about 2 dB between the calculations and the measurements.

It is known from further measurements that the i.f. detector used in this receiver behaves as a linear mean detector to components of signal and noise differing in frequency by less than about 5 kc/s, but that at higher difference-frequencies the behaviour tends towards that of a peak detector, due to the effect of the bias voltage developed across the capacitance of the detector load. A satisfactory mathematical treatment for a practical detector of this type has not been produced.

4. CONCLUSIONS.

The theoretical background of signal/noise ratio improvement has been given, based on an analysis of an idealised linear detector. An application of the theory to a special-purpose receiver, designed for field-strength recording, gives an agreement which is considered reasonable in view of the known differences between the practical and the ideal detector.

In a receiver using post-detector selectivity, such as the one described here, the signal/noise ratio at the detector input is less than unity over a large part of the range of measurable signals; if the noise level in the i.f. amplifier is changed by the addition of an attenuator, the amplitude characteristic of the receiver will be considerably distorted. If the range of measurable signals is to be extended by the use of calibrated attenuators, either these must be inserted at the input to the receiver or a separate calibration will be necessary for each value of attenuation.

5. REFERENCES.

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